

Physical model of the plucking process in the classical guitar

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Abstract

This paper addresses the problem of devising a physical model of the plucking process, here applied to a specific instrument, the classical guitar. We introduce the string equations, and define the body that excites the string (actually the finger) via a set of parameters measured empirically on a real instrument. Then, we evaluate the contribution of the parameters in different phases of the body-string interaction, to produce distinct sounds.

1 Introduction

In the stringed instruments (chordophones) the sound is generated by vibrating strings attached to some resonant system that amplifies and radiates the sound. From the acoustic point of view, we can classify the excitation method, that puts the string in vibration, as:

- a) *striking* (piano)
- b) *bowing* (violin, cello)
- c) *plucking* (guitar, harpsichord)

The sound of a stringed instrument is basically related to the process of string excitation. Many papers have addressed several aspects of the bowing or the striking processes, but up to now the plucking process has not been widely investigated. Some relevant exceptions are the studies on the harpsichord (see, e.g., [7]), and the work in [9], which shares some similarities with the approach presented here (not only because of the reference instrument, the classical guitar). Pavlidou and Richardson describe a one-dimensional model which relies mainly on the frictional force between the fingernail and the string, and solved numerically through the finite differences method. They also sketch an improvement to a two-dimensional model which can include transverse and torsional waves traveling along the string. However, they lack a thorough experimental testing and do not report on the harmonic content of the sound results.

In guitar playing, the sound of each note depends on a number of factors which are related to the player's action on the strings, and not only on the response of the instrument's body. This paper aims at investigating

the physical aspects of the interaction between finger and string in order to create a quantitative model of the touch. The basis for the model is a mechanical representation of the finger based on classical Newtonian equations, solved in the discrete time domain through the *waveguide* or *delay lines* method [10] [8], in contrast with the *finite differences scheme* [2] [3] [9]. Both the methods provide comparable results, but the first one requires a much lower computing time and effort.

We have evaluated the parameters of the model and checked them in various simulated playing situations. The results are in agreement with the common experience of guitarists, and can give a physical insight on the main sound production mechanisms as established by the instrumental technique.

The model of the touch is part of a work aiming at defining a complete physical model of the classical guitar which considers all the factors related to the sound production mechanisms, like the lossy string, the resonator and their interaction. The simulated sound pressure signals presented in this paper result from that model [4].

The paper is organized as follows. In the next section we introduce the basic equations that represent the finger-string interaction and their solution in the discrete domain. The third section is completely devoted to the touch: we list the plausible values where the finger's parameters fall; then we identify the phases of the finger-string interaction (*excitation*, *release*, *dampening*) and we evaluate the contribution of the finger's parameters to the sound qualities during each phase. Finally, we provide some conclusions.

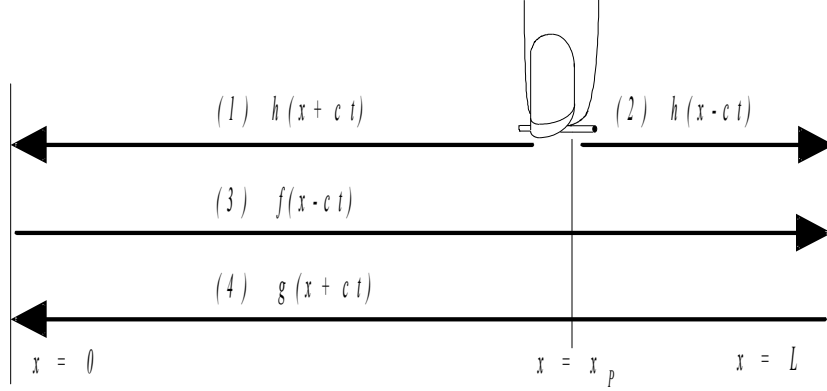


Figure 1: Traveling waves in a plucked string

2 Modeling the action of the finger on the string

The contribution of the touch to the tonal characteristics of the sound is well known to all guitarists. The skilled players can obtain fascinating effects from their instrument by controlling the position of the finger or its action on the strings; only recently some authors [11] [6] have investigated the mechanical aspects of the instrumental technique to correlate the effects that the player can obtain with the resources that he can control.

We can think of a few basic parameters which a guitarist can adjust in order to produce different sounds:

- (a) the *position* of the plucking point along the string,
- (b) the *manner* in which the string is excited,
- (c) the *manner* in which the string is released,
- (d) the *interaction* between the *flesh* or the *nail* and the vibrating string before the attack,
- (e) the *initial displacement* of the string, resulting from the *plucking force*.

While (a) and (e) are described by only one variable (position or force), (b), (c), and (d) are complex phases during which a number of different variables are involved, like the duration of the contact, the physical qualities of the body touching the string (in practice flesh, nail or both), the value of the force and how it is modulated during the contact. All the above considerations lead to the conclusion that the conventional representation of the plucked string (the one with given displacement and zero acceleration at all points before release) is not adequate: in order to reproduce realistically the sound, a model which describes the player-instrument interaction should consider the resources which are under the player's control.

When the string is excited at one internal point, the motion of each element of the string results from the combination of four traveling waves [5] [8]:

- (a) the wave originated at the plucking point and traveling towards the nut,
- (b) the wave originated at the plucking point and traveling towards the bridge,
- (c) the wave reflected from the nut and traveling towards the bridge,
- (d) the wave reflected from the bridge and traveling towards the nut.

In the plucking point (located at $x = x_p$), the displacement of each point along the string is given by

$$y(x, t) = f(x - ct) + g(x + ct) + h(x + ct) \quad \text{for } x < x_p$$

$$y(x, t) = f(x - ct) + g(x + ct) + h(x - ct) \quad \text{for } x > x_p \quad (1)$$

$$y(x_p, t) = f(x_p - ct) + g(x_p + ct) + h(x_p, t) \quad \text{for } x = x_p$$

which are a particular form of the D'Alembert solution of the wave equation. The *perturbation wave* $h(x, t)$ is only related to the excitation mechanism, while the progressive waves $f(x, t)$ and $g(x, t)$ are only related to the impedances of the supports.

The force applied to the plucking point is balanced by the internal components of the string tension applied to an element dx . For small string displacements the equilibrium condition yields

$$F(t) = F_x \left[\frac{\partial y}{\partial x} \right]_{x=x_p^-} - \left[\frac{\partial y}{\partial x} \right]_{x=x_p^+} = F_x D \quad (2)$$

where F_x is the string tension and D is the *difference of increments* at each side of the plucking point.¹

Now let us assume that the action on the string is applied by a body (actually the player's finger) defined by given mass M_d , stiffness K_d , damping coefficient R_d [1]; the force acting on the string at the plucking point $F(t)$ is related to the force $F_0(t)$ available from the finger by

¹ Eq. (2) is also valid when the excitation is distributed along a segment, provided that the increments are calculated at the extremes of the segment itself.

$$F(t) = F_0(t) - (M_d + \mu \Delta) \frac{\partial^2 y}{\partial t^2} - R_d \frac{\partial y}{\partial t} - K_d y \quad (3)$$

Note that here we assume for the finger a linear compression law and a simple fluid damping coefficient ; we claim that the quality of the sound resulting from the finger-string interaction is mostly related to the manner in which the force is applied or released, and to the values of the parameters controlled by the player; the (possible) non linearities of these parameters are a second order effect which, we believe, can be neglected.

In order to obtain a solution in the discrete time domain, Equations (1-3) are sampled in time and space; the ratio between the spatial sampling interval Δ and the time step T is equal to the string longitudinal velocity c . The sampled form of Eq. (1) and (2) yields the function D.-

$$D = \frac{2}{\Delta} [h(n_p, m) - h(n_p, m-1)] + \frac{2}{\Delta} w(n_p, m) - \frac{1}{\Delta} [w(n_p, m-1) + w(n_p, m+1)] \quad (4)$$

with $w(n, m) = f(n, m) + g(n, m)$.

Replacing in Eq. (2) the sampled form of Eq. (3) gives $h(n_p, m+1)c_0 = h(n_p, m-1)c_1 +$

$$[h(n_p, m) + w(n_p, m)]c_2 + w(n_p, m-1)c_3 + w(n_p, m+1)c_4 + F_0 c_5 \quad (5)$$

by means of which one can calculate the value of the perturbation in the plucking point, given the physical characteristics of the finger and the value of the excitation. The constants in Eq. 5 are given by

$$\begin{aligned} c_0 &= 1 + \frac{R_d T}{2(M_d + \mu \Delta)} \\ c_1 &= -1 + \frac{R_d T}{2(M_d + \mu \Delta)} + \frac{2F_x T^2}{\Delta(M_d + \mu \Delta)} \\ c_2 &= 2 - \frac{2F_x T^2}{\Delta(M_d + \mu \Delta)} - \frac{K_d T^2}{(M_d + \mu \Delta)} \\ c_3 &= -1 + \frac{R_d T}{2(M_d + \mu \Delta)} + \frac{F_x T^2}{\Delta(M_d + \mu \Delta)} \\ c_4 &= -1 - \frac{R_d T}{2(M_d + \mu \Delta)} + \frac{F_x T^2}{\Delta(M_d + \mu \Delta)} \\ c_5 &= \frac{T^2}{(M_d + \mu \Delta)} \end{aligned} \quad (6)$$

As a particular case it follows from Eq. (5) and Eq. (6) that, when the string oscillates freely, the perturbation function is given by

$$h(n_p, m+1) = h(n_p, m-1) \quad (7)$$

that is, the perturbation wave at the plucking point has the same value every two time samples. The excitation

at the plucking point (eq. 5) and the motion of the progressive waves along the string (eq. 1) define completely the motion of the string.

While the most natural structure for representing the motion of the waves along the string would require three arrays (one for each of the waves $h(x, t)$, $f(x, t)$, $g(x, t)$), we have developed the two arrays structure shown in fig 2: at each spatial sample n and at the time sample m the string displacement is

$$y(n, m) = |Upper[n] + Lower[n]|_m \quad (8)$$

The *reflection* coefficients account for reflections of the waves at the extremes of the string.

3 The parameters of the touch

During each step of its action on the string (*excitation*, *release*, *dampening*) the player is able to control the parameters of the touch. On the basis of a few empiric measures we can find a range of plausible values for each of the parameters controlled by the player.

The strength of the plucking force is related to the displacement of the string before release, and hence to the intensity of the sound pressure. In normal playing conditions the displacement ranges approximately between 1 mm (*piano*) and 5 mm (*forte*), which corresponds to a force ranging between 1 N and 5 N.

The mass of the finger M_d can't be measured directly, but we assume that the player *controls* the mass by *loading* the string with the fingertip, i. e. by relaxing or strengthening the tip while playing; quantitatively the mass of the finger lies between that of the electric guitar plectrum (about 0.2g) and that of the piano hammer (about 10g); we assume that it ranges between 0.5 g and 3 g.

The phalanx touching the nail consists of a hard and stiff body, of horny character (the *nail*), which basically draws the string away from its initial position, and an elastic and soft body (the *flesh*) which basically damps the string's motion by absorbing its energy. By *rotating* the whole hand from the wrist or *bending* the tip joint, the guitarist controls both the stiffness and the damping of the finger while touching the string.

The term K_d (Eq. 3) represents the stiffness of the finger. The stiffness of the nail probably is not far away from that of the plectrum of an electric guitar. We found that the plausible value of the finger stiffness lies between $K_d = 0$ (when only the flesh draws the string) and $K_d = 3 \cdot 10^3$ N / m (when the effect of the nail prevails).

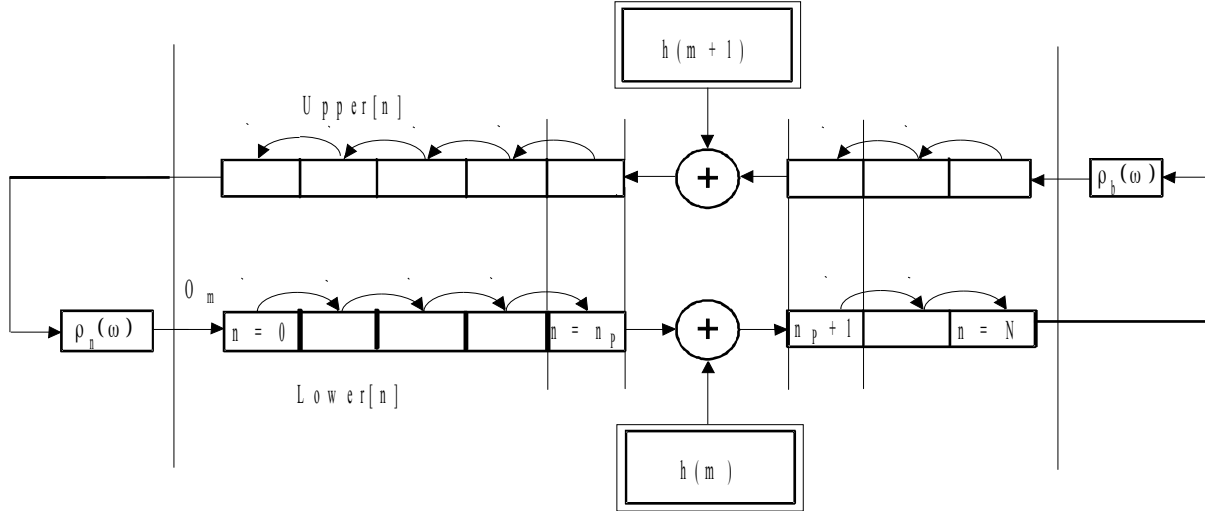


Figure 2. The two-array model of the plucked string

The damping coefficient $r_d = R_d / R_c$, normalized to the string's *characteristic resistance*, can be evaluated from the decay time of the oscillation of the string loaded by the finger, when no force is applied. With a set of values for mass and stiffness which can be considered as typical ($M_d = 0.5$ g, $K_d = 0$) we can assume that r_d ranges normally between 2 and 5, while only in case of heavy dampenings it can reach 8.

Now we analyze in detail the three phases of finger-string interaction: excitation, release, dampening.

3.1 String excitation

According to the physicist's definition, the usual action of exciting a guitar string is always one of *plucking* (as for the quill of a harpsichord) as opposed to *striking* (as for the piano's hammer action). In plucking, the fingertip is considered initially at rest on the string while in striking it arrives at the string with a certain speed. Now, in the case of the guitar, there is no difference in these two actions from the string's viewpoint, in that the string is firstly *driven aside* and then *released*, and so the sound is *unlikely to be affected by the speed* at which the finger approached the string. Certain sounds are produced by a quick *impulse* of the fingertip on the string (hammer-like action, or *apoyando*); in other cases the applied force grows gradually during the attack, giving the string a gentler ride (*tirando*). It is worth mentioning that the *noise* and the *effect of separation* between sounds due to the contact of the tip with the string are both relevant for the character of the guitar's sound. In our model the first action (*apoyando*) is described by applying in (Eq. 5) a stepwise force $F(t) = F_0$ and maintaining it for a short time before the release; in the

second case (*tirando*) the force is assumed to increase linearly during the application time, up to its maximum value:

$$F(t) = F_0 * \frac{t}{t_{appl.}} \quad (9)$$

Fig. 3 shows the Fast Fourier Transform of the sound pressure signal generated by the model in the two different conditions: in (a) the string is driven aside gradually during 20 ms (linear attack), while in (b) the force is applied stepwise and maintained for 8 ms. It is evident that in (a) the fundamental prevails over the other partials.

The damping coefficient r_d tends to smooth the transient oscillations occurring during the plucking, mainly when the force is applied stepwise. The reaction of the string on the tip is higher when the mass is high, provided the damping coefficient is low (see fig. 4). Also, when the mass is high, higher partials prevail.

However, when the damping coefficient is high (e.g. $r_d=6$), the influence of the tip mass during the excitation phase is low (see fig. 5) and we can observe a similar distribution of the partials.

In summary, when the force increases linearly during the excitation time, the parameters of the fingertip have little influence on the sound, because it doesn't matter how the string comes to its final displacement provided the excitation is long enough for the transients to extinguish. If the string is pulsed, the string-tip interaction is more evident when the mass is high; moreover, the lower is the damping coefficient, the higher is the string reaction. In the former situation (linear application) the fundamental prevails, while in the latter (step application) the higher partials prevail.

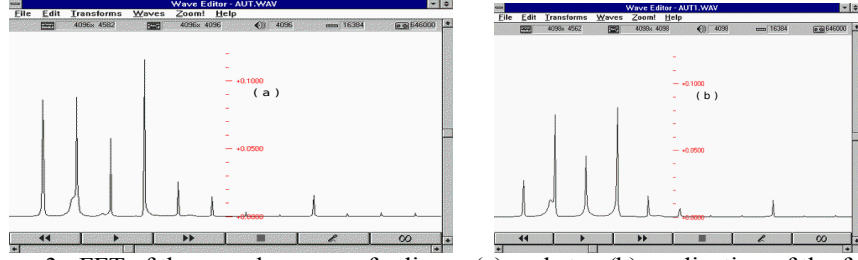


Figure 3. FFT of the sound pressure for linear (a) and step (b) application of the force.

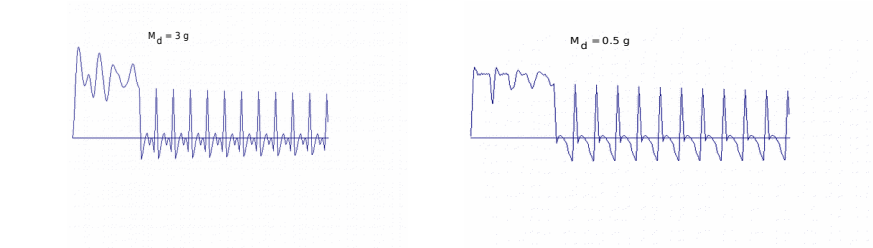


Figure 4. Displacement at plucking point with two values of the tip mass for $r_d = 1$.

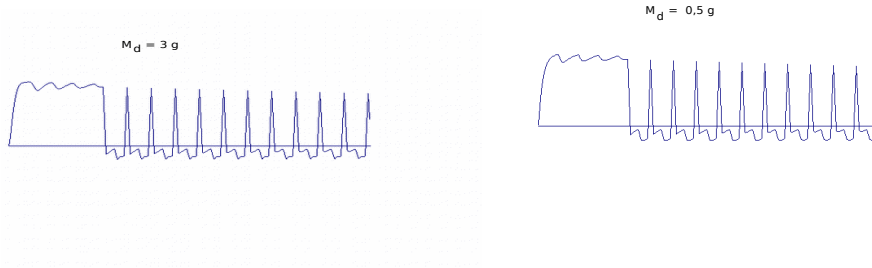


Figure 5. Displacement at plucking point with two values of the tip's mass for $r_d = 6$.

3.2 String release

When the force which displaces the string is suddenly removed, the string tension, no longer balanced by the external force, acts at the plucking point and drives into motion all the points between the terminations. The manner in which the string is released is a crucial factor for the quality of the sound. The sudden variation of displacement excites the higher modes while, conversely, these modes can be suppressed by releasing the string gradually. The guitarist controls the release by presenting the tip in such a way that the string can *slide* over the flesh (or the nail) for a certain time before leaving it; during this *release time* the force which was keeping the string in place is removed

gradually, but the finger is *still in contact* with the string and interacts with it by smoothing its initial motion. This interaction is consequently dominated not only by the *time* during which the string is released, but also by the *physical properties* of the tip.

In order to transfer these ideas into our model we make the assumption that the force is removed gradually during the release:

$$F(t) = F_{Max} * \left[1 - \frac{t - t_{appl}}{t_{rel}} \right] \quad (10)$$

and that the string can slide over the tip for a shorter time than the cycle of the string oscillation, in order to

avoid big energy losses. This limits the release time to a few milliseconds.

To describe the release, it is plausible to introduce a new set of mechanical parameters for the fingertip, respectively M_{d_r} , K_{d_r} , r_{d_r} , which again one can correlate to the instrumental technique: 1) if the release is performed by using mainly the nails, K_{d_r} will be

higher; 2) a lower mass M_{d_r} is associated with a more *relaxed* action; 3) the normalised damping coefficient r_{d_r} will be associated with the absorbing properties of the flesh during release. Now we present a few typical situations, to check the consistency of the model results with the ideas discussed above.

Fig. 6 illustrates the influence of the release time: in (a) the release is instantaneous, in (b) the force is removed gradually ($t_{rel} = 4$ ms); see the strong influence of the release time on the higher partials.

In Fig. 7 (a) $K_{d_r} = 0$, while in (b) $K_{d_r} = 5 \cdot 10^3$ N / m. Clearly in (b), where the intervention of the nail while releasing is simulated, the higher string modes are more important than in (a); it is well known that

the use of the nails gives the sound a particularly clear - sometimes metallic - character.

The damping coefficient prevents the formation of higher modes: fig. 8 shows the distribution of harmonics for (a) $r_{d_r} = 5$ (high damping coefficient) and (b) $r_{d_r} = 0$ (no damping): the higher is the damping and the lower is the amplitude of high partials.

In summary, by modeling different situations which can occur during release, we have found that a sudden release excites higher string modes, while a gradual decrease of the force points out the fundamental. A high value of stiffness supports the excitation of higher modes, while a high damping coefficient prevents the excitation of higher modes.

These findings, which are basically in agreement with the common experience on the instrument, can give a (partial) physical explanation of the reasons why a skilled guitarist can obtain so different and fascinating sounds from his instrument.

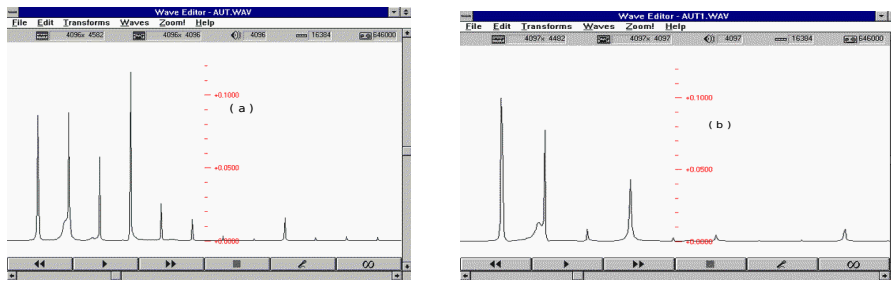


Figure 6. FFT of the simulated sound pressure in two release time conditions.

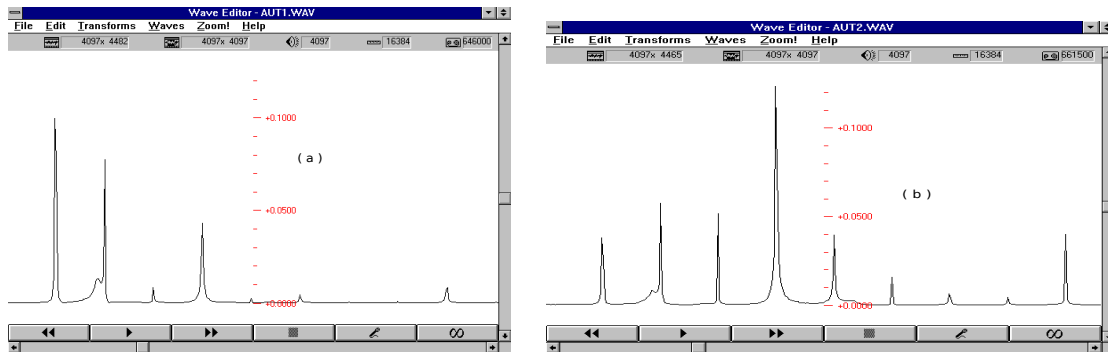


Figure 7. The influence of the tip stiffness during release

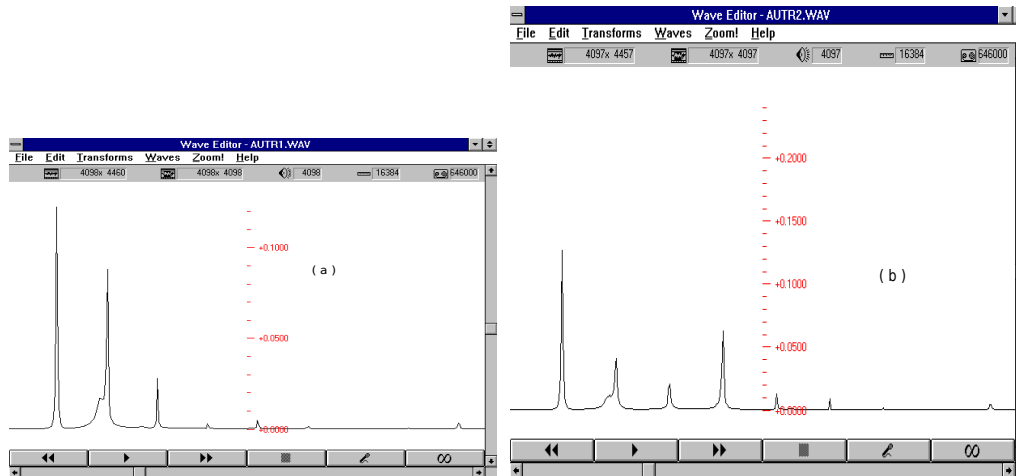


Figure 8. The influence of the tip's damping coefficient during release

3.3 String damping

To control the duration of a note, the player has to extinguish the oscillation of the string at the right time; similarly, s/he must damp the oscillation of a vibrating string (before plucking it again) in order to minimize the noise produced by the contact of the string with the fingertip. For this purpose, generally the player touches the string with the flesh *without exerting any force* on it, that is *without loading the string with the weight of the hand*. This action is modeled by setting to zero, in the perturbation function, all the finger's parameters except the normalized damping coefficient. Fig. 9 demonstrates the effect on the string motion at the plucking point due to two damping coefficients, respectively (a) $r = 4$ and (b) $r = 1$; once again $r = R / R_c$, where R_c is the string characteristic resistance.

The envelope of the peaks decays exponentially; the time constant is higher in (b) than in (a) where the damping effect is lower.

The model allows the simulation of consecutive notes played on the same or on different strings; in fig. 10 the same note is played twice on the same string. Here damping is applied between the first and the second note; the motion of the string during the second attack is a combination of the first, not yet extinguished, oscillation, with the displacement given by the plucking force. The effect on the sound does not seem so relevant (except for the audible rattle due to the residual oscillation during the second attack); but, if the attack is short, the residual oscillation can modify the displacement of the second note and, consequently, does have a noticeable effect on the sound. This effect, which is easily reproducible and explained with the aid of the model, is commonly experienced by guitarists.



Figure 9. Dampening the string with two different damping coefficients



Figure 10. String damping between two notes

4 Conclusions

We have developed a model of the plucking which can be considered a first approach to a very complex and not yet well understood problem; nevertheless, it covers the fundamental aspects of the tone production mechanism in the classical guitar, related to the interaction between the string and the player.

To understand the mechanisms of the player's action, it was necessary to gain an insight into items traditionally belonging to the instrumental technique, in order to express them in terms of the underlying physical principles.

The mechanical model of the finger is based on a simple linear representation (mass, stiffness, damping coefficient); although the hypothesis of linearity does not fit exactly the reality, it has empirically proven to be accurate enough: the basic idea is that the player modifies the parameters of the finger, by *loading* the string with the weight of the hand (*mass*), or by playing with the nails rather than with the flesh alone (*stiffness*), or finally by smoothing the motion of the string through the absorbing properties of the flesh (*damping coefficient*). This is one important difference between the piano's hammer (a typically non linear exciter) and the guitarist's finger. For each of the finger parameters we can define a *set* of plausible values (rather than a *single value*). According to our description of the tone production mechanism, the conceptually infinite variety of sounds which the player can obtain depends on the combination of a relatively limited number of parameters.

We have also investigated the *perturbation wave* (the one which originates from the action of the exciter on the string); we have found a mathematical formulation which is compatible with the basic algorithm based on the delay line representation to follow the waves traveling along the string. Our formulation of the perturbation wave is suitable to describe the various stages which we have individuated in the plucking action (*excitation*, *release*, *damping*), and the different sets of finger parameters applied at each stage.

Finally we have presented some typical situations in order to demonstrate the consistency of the results from the model with the arguments of the instrumental technique. Among the interesting features of the model, we would like to mention the possibility of reproducing different attacks (*apoyando* and *tirando*) in contrast with the static and conventional description of the plucked string (the one with an initial displacement and zero velocity at all points), and the possibility of modeling different release conditions, mostly relevant for the quality of the sound and the transients occurring when playing consecutive notes.

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